

ESTIMATING THE RELIABILITY OF UNRECOVERABLE ONE-COMPONENT HOMOGENOUS SYSTEMS WITH GRADUATE FAILURES*

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A b s t r a c t: In this paper we consider a multi-state one component systems with linearly ordered levels of performance. It is assumed that at a given moment of time the system can fail down only for one level with constant transition intensities. Under these assumptions we give the explicit transition probabilities function that we use to estimate the transition intensities. There are two types of systems for which we estimate the parameters, the systems that are under complete control and the systems for which we can observe only the time of total failure.

Key words: multi-state system; reliability; estimators

INTRODUCTION

The multi-state systems are systems that can work at different levels of performance. The basic concepts about these types of systems are given by Natvig [2]. Most of the multi-state reliability literature analyze the system reliability using the system structure, without considering the work of the system as a process during the time, as in [4], [5], [8]. One of the rare papers that deal with probabilistic aspects of modeling system reliability is [1], where the authors analyze repairable systems and the failure probability is given in terms of Laplace transform. Also, there are very rare papers in which the reliability is

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estimated using the reliability data. Such paper is [6], where the authors analyze monotone systems with multi-state components in which minimal path and cut sets of any level are equal, and the estimation of reliability is based on the number of periods.

In this paper we assume that the multi-state components are independent and the random variable “one step transition time” of each component has exponential distribution. It means that the components are homogenous with unknown failure intensities. The main goal is to analyze the work of the system in order to determine estimators for the unknown failure intensities. Since we assume that the components are independent, it is enough to observe the work of a single component. We will give three types of estimators for the unknown parameters for a given one-component homogenous multi-state system.

SYSTEM DESCRIPTION

As we said before, the type of systems we are interesting at are multi-state systems with one component. Let us suppose that the set of all states of the system, i.e. the state set of the system is finite, $E = \{0, 1, \dots, M\}$. We also assume that the states are linearly ordered, and higher state means better performance. Formally, for $i < j$, the system which is in the state i works with lower quality than the system which is in the state j . The state 0 is the state of total failure of the system and the state M is the perfect state of the system. The system can gradually fail over time and the only possible failure transitions are from state i to state $i - 1$, which will be called one step transitions. We are not interested in recovering the systems, so we suppose that the system operates without any improving until its total failure. Each transition is independent from the previous transitions; it only depends on the last state of the system. This means that the one step transitions are homogenous Markov transitions. Such a system is known as unrecoverable one-component homogenous system with graduate failures.

In order to estimate the reliability function of the system we need to estimate the intensities of failure which are usually unknown. Therefore, we will consider a few ways for estimating these intensities.

There are several settings for observing the data. First, it is possible to observe the exact transition times from one to another state of the system, i.e. to have a system which is under control. On the other hand, we can observe only the time of total failure of the system. Also we can have a system for which we

know that at the time t it works at level i , but we do not know the exact time when the system entered that state.

As a data used to estimate the reliability function, we will take the time at which the system is observed together with the state at which it is found at that time. There are two types of data, censored and uncensored. When the data represent the exact transition times, we have uncensored data. If we do not know the exact time when the system fell below the level j , but we know that at the time t the state of the system was higher than j , we talk about right censored data. If we know that at the observed time t the system is in some state lower than j , but the exact transition time is not known, then the time t is left censored data. If we know that the transition at level j occurred in some time interval, we have interval censored data. Note that for an unrecoverable system, the time t at which the system operates at level j represents left censored data for level j and right censored data for level $j - 1$.

In this paper we will analyze the methods we can use to estimate the reliability function for different data types mentioned above. As a starting point we will determine the transition and reliability functions for such system, and after that, using this functions, we will find the estimators of the unknown parameters.

TRANSITION AND RELIABILITY FUNCTIONS

At the beginning of this section we will find the transition probability matrix for one-component homogenous systems with graduate failures and next we will use it to determine the reliability function for this type of systems. First we give some definitions:

Given two different states of the system, i and j let $p_{i,j}(t)$ be the probability that the system will transit from state i to state j in one step for time t and $q_{i,j}$ be the intensity of that transition. Using these probabilities we create the *transition probability matrix* [3].

$$P(t) = (p_{i,j}(t))_{n \times n}. \quad (1)$$

where, the function $p_{i,j}(t) = P(X(t_0 + t) = j \mid X(t_0) = i)$. The function $p_{i,j}(t)$ is called *transition probability* from state i to state j . In order to obtain the transition probabilities and reliability function we will use the Kolmogorov system of differential equations:

$$P'_{i,j}(t+h) = \sum_{k \in E} p_{i,k}(t) q_{k,j} . \quad (2)$$

The matrix form of the Kolmogorov system of differential equations is given by:

$$P'(t) = P'(t)Q , \quad (3)$$

where $P(t)$ is the transition probability matrix and $Q = (q_{ij})_{n \times n}$ is the intensity matrix. We have that $p_{ii}(0) = 1$ and for $i \neq j$, $p_{ij}(0) = 0$. So, the initial condition is $P(0) = I$. It follows that there is a unique solution of the Kolmogorov system [3]:

$$P(t) = e^{Qt} \quad (4)$$

Using the Taylor series the last equation can be written as:

$$P(t) = \sum_{n=0}^{\infty} \frac{Q^n t^n}{n!} . \quad (5)$$

All of the intensities of the system we deal with are different.

Therefore, the intensity matrix is equal to:

$$Q = \begin{bmatrix} 0 & 0 & 0 & \dots & 0 & 0 & 0 \\ q_1 & -q_1 & 0 & \dots & 0 & 0 & 0 \\ 0 & q_2 & -q_2 & \dots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & -q_{M-1} & 0 & 0 \\ 0 & 0 & 0 & \dots & q_{M-1} & -q_{M-1} & 0 \\ 0 & 0 & 0 & \dots & 0 & q_M & -q_M \end{bmatrix} \quad (6)$$

Theorem 1. If $q_{i,i-1} \neq q_{j,j-1}$ for $i \neq j$ then

$$P(t) = SBS^{-1} , \quad (7)$$

where B is a diagonal matrix such that $b_{1,1} = 1$, $b_{i,i} = e^{-q_{i-1}}$ and $S = (s_{i,j})_{n \times n}$ and $S^{-1} = (\hat{s})_{n \times n}$ are given with

$$s_{i,j} = \begin{cases} 0, & i < j \\ \prod_{k=i+1}^M \frac{q_k - q_j}{q_k}, & i \geq j > 0 \\ 1, & j = 0 \end{cases} \quad (8)$$

and

$$\hat{s}_{j,k} = \begin{cases} \frac{\prod_{i=k+1}^m q_i}{\prod_{i \geq k, j \neq i} (q_i - q_j)}, & j, k > 0 \\ \frac{\prod_{i \neq j} q_i}{\prod_{i \neq j} (q_i - q_j)}, & k = 0, j > 0 \\ 1, & j = k = 0 \end{cases} \quad (9)$$

Proof. The eigenvalues of Q are $0, -q_1, -q_2, \dots, -q_M$ and they are all different. So, the corresponding eigenvectors of Q are independent. Let A be the diagonal matrix of the eigenvalues of Q , i.e. $a_{1,1} = 0, a_{i,i} = -q_{i-1}$ for $i \neq 1$ and $a_{i,j} = 0$, for $i \neq j$, and let S be the matrix whose columns are corresponding eigenvectors. Since all columns of S are independent, the matrix S^{-1} exists, and we have that Q is equal to:

$$Q = SAS^{-1}$$

and, the matrix Q^n is equal to:

$$Q^n = SA^nS^{-1}$$

Now, for the transition probability matrix we obtain:

$$P(t) = \sum_{n=0}^{\infty} S \frac{A^n t^n}{n!} S^{-1} = S \left(\sum_{n=0}^{\infty} \frac{A^n t^n}{n!} \right) S^{-1} = SBS^{-1},$$

where B is a diagonal matrix such that $b_{1,1} = 1$ and $b_{i,i} = e^{-q_{i-1}}$.

Technical details for obtaining (8) and (9) are given in [7]. $\square$

Using the Theorem 1, we can obtain the function $p_{i,j}(t)$.

Theorem 2.

$$p_{i,j}(t) = \begin{cases} \sum_{k=j}^i \frac{\prod_{m=j+1}^i q_m}{\prod_{j \leq m \leq i, m \neq k} (q_m - q_k)} e^{-q_k t}, & i > 0 \\ 1 - \sum_{k=j}^i \prod_{1 \leq j \leq i, j \neq k} \frac{q_j}{q_j - q_k} e^{-q_k t}, & i = 0 \end{cases} \quad (10)$$

Next in this section we determine the reliability function for the systems under consideration.

Lemma 1. The reliability function for the level j , when the system starts operating at the level i , for $i > j$ is:

$$R_{i,j}(t) = \sum_{k=1}^i \prod_{j+2 \leq m \leq i, m \neq k} \frac{q_m}{q_m - q_k} e^{-q_k t} \quad (11)$$

Proof.

$$\begin{aligned} R_{i,j}(t) &= P(T_{i,j} > t) = \sum_{r=j+1}^i p_{i,r}(t) = 1 - \sum_{r=0}^j p_{i,r}(t) \\ &= \sum_{k=1}^i \prod_{j+2 \leq m \leq i, m \neq k} \frac{q_m}{q_m - q_k} e^{-q_k t}. \end{aligned} \quad \square$$

By the Lemma 1, for the reliability function we have:

Theorem 3. Let $\pi = (\pi_0, \pi_1, \dots, \pi_M)$ be the initial probability of the system. Then the reliability function for j^{th} level is equal to:

$$R_j(t) = \sum_{i=0}^M \pi_i R_{i,j}(t), \quad (12)$$

where $R_{ij}(t)$ are given by Lemma 1.

Corollary 1. When the system starts operating at the perfect state, $\pi = (0, 0, \dots, 1)$, the reliability function is equal to $R_{M,j}(t)$.

ESTIMATING THE PARAMETERS OF THE SYSTEM WHICH IS UNDER COMPLETE CONTROL

We say that the system is under complete control if it is possible to determine the operating state at any moment of its operation. Let t_0 be some fixed time. Suppose that N identical systems start operating at the time $t = 0$ and at the time $t = t_0$ they are checked in order to verify their state. By X we will denote the random variable “the state of the system at the time $t = t_0$, under assumption that it was at the perfect state at time $t = 0$ ”.

Next Theorem gives us an estimator of the failure intensities of such model.

Theorem 4. Let $X(t)$ be the random variable “the state of the system at the time t , under assumption that it started operating at the time 0 at level M ” and $x_1, x_2, \dots, x_N$ be a random sample of the random variable $X(t_0)$. By N_i we denote the frequency of the value i , $i = 1, \dots, M$. Then the maximum likelihood estimator of the intensity vector $\mathbf{q} = (q_1, q_2, \dots, q_M)$ is obtained solving the system

$$p_{M,i}(t_0) = \frac{N_i}{N}, \quad i = 0, \dots, M, \quad (13)$$

where $p_{M,i}(t_0)$ are given by (10).

Proof. Let $F_i(t) = P(X(t) > i)$ i.e. $F_i(t)$ is the reliability of the state i . By $S_i(t)$ we will denote the probability that the system that started operating at time 0 at level M , at the time t is found at some level lower than i . We have

$$F_i(t) = P(X(t) \geq i) = \sum_{j=i}^M p_{M,j}(t), \quad i = 0, \dots, M$$

$$S_i(t) = 1 - F_i(t), \quad i = 0, \dots, M$$

The likelihood function can be constructed in the following manner: if at the time t the system works with level higher than i , we take the function $F_i(t)$, otherwise, we take the function $S_i(t)$. Suppose that K_i of the random sample x_j , $j = 1, \dots, N$ have a values higher than i . Then the likelihood function is equal to:

$$L(q, (t, t, \dots, t)) = (F_i(t))^{K_i} (1 - F_i(t))^{N - K_i}, \quad i = 0, \dots, M$$

Now, for all $i = 1, \dots, M$

$$l(q, (t, \dots, t)) = K_i \ln(F_i(t)) + (N - K_i) \ln(1 - F_i(t))^{N-K_i}.$$

By differentiation we obtain:

$$K_i \frac{F'_i(t)}{F_i(t)} + (N - K_i) \frac{-F'_i(t)}{1 - F_i(t)} = 0 \Leftrightarrow \frac{K_i}{N} = F_i(t)$$

Using the fact that $p_{M,i}(t) = F_i(t) - F_{i-1}(t)$ we have:

$$p_{M,i}(t_0) = F_i(t_0) - F_{i-1}(t_0) = \frac{K_i - K_{i-1}}{N} = \frac{N_i}{N}. \quad \square$$

Each of the functions $p_{M,i}(t_0)$ depends of $M - i + 1$ parameters, the parameters can be estimated stepwise starting from

$$p_{M,M}(t_0) = e^{-q_M t_0} = \frac{N_M}{N}.$$

Using this equality we can easily estimate the parameter q_M . Substituting the obtained values in the equation for $P_{M,M-1}$ and by solving it, we will estimate the parameter q_{M-1} . Continuing on the same way, we will obtain all unknown intensities. In the most of the cases the equations can not be solved exactly, so we must use some approximating methods.

ESTIMATING THE PARAMETERS OF THE SYSTEM WHICH IS NOT UNDER COMPLETE CONTROL

Sometimes it is not possible to determine the state of the system while it is in a working condition. In this section we assume that all intermediate deterioration states, until the system fails to level 0, can not be determined, but the time of total failure can be observed. It is also assumed that the number of states the system is known. The problem we regard here is to find estimators for the unknown intensity parameters under such settings.

Theorem 5. Let T be the random variable “time to total failure of the system” and $t_1, t_2, \dots, t_N$ be a random sample of T . Then the maximum likelihood estimators for the failure intensities are the solutions of the system:

$$\frac{N}{\hat{q}_m} = \sum_{k=1}^N \frac{1}{\sum_{i=1}^M \frac{e^{-\hat{q}_i t_k}}{\prod_{j \neq i} (\hat{q}_j - \hat{q}_i)}} \left(\sum_{i \neq m} \frac{e^{-\hat{q}_i t_k}}{(\hat{q}_j - \hat{q}_i) \prod_{j \neq i} (\hat{q}_j - \hat{q}_i)} + \right. \\ \left. + \sum_{i \neq m} \frac{e^{-\hat{q}_i t_k}}{(\hat{q}_j - \hat{q}_i) \prod_{j \neq m} (\hat{q}_j - \hat{q}_m)} + \frac{t_k}{\prod_{j \neq m} (\hat{q}_j - \hat{q}_m)} \right). \quad (14)$$

for $m = 1, \dots, M$.

Proof. Using (10) for the distribution and density function we have:

$$F(t) = p_{M,0}(t) = P_M(T_0 < t) = 1 - \sum_{i=1}^M \prod_{j \neq i} \frac{q_j}{(q_j - q_i)} e^{-q_i t}$$

$$f(t) = F'(t) = \left(\prod_{i=1}^M q_i \right) \sum_{i=1}^M \frac{e^{-q_i t}}{\prod_{j \neq i} (q_j - q_i)}$$

The likelihood function is:

$$L(t, q) = \prod_{k=1}^N f(t_k) = \left(\prod_{i=1}^M q_i \right)^N \prod_{k=1}^N \sum_{i=1}^M \frac{e^{-q_i t_k}}{\prod_{j \neq i} (q_j - q_i)}.$$

Taking logarithm we have

$$l(t, q) = N \sum_{j=1}^M \ln(q_j) + \sum_{k=1}^N \ln \left(\sum_{i=1}^M \frac{e^{-q_i t_k}}{\prod_{j \neq i} (q_j - q_i)} \right).$$

Now the ML estimators for q_m are obtained by solving the $\{dl/dq_m = 0 \mid m = 1, \dots, M\}$, i.e. the system

$$0 = \frac{N}{q_m} + \sum_{k=1}^M \frac{\frac{d}{dq_m} \left(\frac{\sum_{i=1}^M e^{-q_i t}}{\prod_{j \neq i} (q_j - q_i)} \right)}{\sum_{i=1}^M \frac{e^{-q_i t}}{\prod_{j \neq i} (q_j - q_i)}},$$

for $m = 1, \dots, M$. Collating this equation it can be rewritten in the form (14).

□

It can be observed that the system in Theorem 5 is symmetric with respect to each of the parameters q_i , $i = 1, \dots, M$, which means that the permutation of some of the parameters will result with the same system. Therefore, there is more than one solution, but all of them are permutations one each other. Since our interest is concentrated only on the total failure, it does not matter which one of these solutions will be chosen.

At this moment we will take a time to give a procedure for finding an ML estimator for model with additional unknown parameter, the number of the states of the system. Suppose that except the q_i , M is also unknown. How we can find an ML estimator for the parameter $(\mathbf{q}, M)$? One procedure is the following. First let us define parameter $\mathbf{q}^M$ as a parameter $\mathbf{q}$ under assumption that the state set of the system is $\{0, 1, 2, \dots, M\}$, $M \in \mathbf{N}$. The algorithm for finding the ML estimator is the following:

1. Initialize $l(\mathbf{q}^0, 0, \mathbf{t}) = \infty$
2. Starting from $M = 1$ estimate the parameter $\mathbf{q}^M$ and calculate the likelihood function $l(\mathbf{q}^M, M, \mathbf{t})$.
3. If $l(\mathbf{q}^M, M, \mathbf{t}) > l(\mathbf{q}^{M-1}, M-1, \mathbf{t})$, then stop and return $\mathbf{q}^{M-1}, M-1$.

It is obvious that the maximum likelihood estimators are very complex and it is hard to use them for estimating the parameters. For that reason next in this section we give an estimator obtained by the method of moments.

Theorem 6. Let T be the random variable “time to total failure of the system” and $t_1, t_2, \dots, t_N$ be a random sample of T . Then the estimator obtained by the method of moments for the failure intensities is the solution of the system:

$$k! \sum_{(k_1, k_2, \dots, k_M) \in V_M} \prod_{i=1}^k \frac{1}{q_i^{k_i}} = \frac{\sum_{i=1}^M t_i^k}{N}, \quad k = \overline{1, M}, \quad (15)$$

where $V_{M,k} = \{\mathbf{r} = (r_1, r_2, \dots, r_M) \mid r_1 + r_2 + \dots + r_M = k\}$.

Proof. Let T_i , $i = 1, \dots, M$ be the random variable "transition time from state i to state $i - 1$ ". Then we have

$$T = T_1 + T_2 + \dots + T_M$$

Since the random variable T_i , $i = 1, \dots, M$ has an exponential distribution, the k -th moment of T_i is

$$E(T_i^k) = \frac{k!}{q_i^k}.$$

The k -th moment of T is

$$E(T^k) = E\left(\sum_{i=1}^M T_i\right)^k = E\left(\sum_{i=1}^M \frac{k!}{r_1! r_2! \dots r_M!} T_1^{r_1} T_2^{r_2} \dots T_M^{r_M}\right).$$

where $(r_1, r_2, \dots, r_M) \in V_{M,k}$. Since the random variables T_i are independent,

$$\begin{aligned} E(T^k) &= \sum_{i=1}^M \frac{k!}{r_1! r_2! \dots r_M!} \prod_{i=1}^M E(T_i^{r_i}) = \sum_{i=1}^M \frac{k!}{r_1! r_2! \dots r_M!} \prod_{i=1}^M \frac{r_i!}{q_i^{r_i}} \\ &= k! \sum_{i=1}^M \prod_{i=1}^M \frac{1}{q_i^{r_i}}. \end{aligned}$$

On the other hand, the k -th moment calculated from the data is $\frac{\sum_{i=1}^N t_i^k}{N}$, so the unknown parameters can be estimated using the system (15).

□

As in the case of the ML estimators, the obtained equations are symmetric with respect to the parameters q_i , $i = 1, \dots, M$. So, again there are multiple solutions, that are permutations of each other.

Example 1. Consider a model of one-component homogenous system with graduate failure for which $q_j = jq$ for some constant q . Finding ML estimator is more complicated using Theorem 6 then taking the procedure used in the proof of the theorem. In this case there is only one unknown parameter, and the function $F(t)$ has the following form:

$$\begin{aligned} F(t) &= 1 - \sum_{i=1}^M \left(\prod_{j \neq i} \frac{jq}{q(j-i)} \right) e^{-iqt} = 1 - \sum_{i=1}^M \frac{\prod_{j \neq i} j}{\prod_{j \neq i} (j-i)} e^{-iqt} \\ &= 1 - \sum_{i=1}^M \frac{M!}{(i-1)!(M-i)!(-1)^{i-1}} e^{-iqt} \\ &= 1 + \sum_{i=1}^M \frac{(-1)^i M!}{i!(M-i)!} e^{-iqt} = (1 - e^{-qt})^M. \end{aligned}$$

The density function is $f(t) = Mqe^{-qt}(1 - e^{-qt})^{M-1}$, so

$$L(t, q) = M^N q^N e^{-q \sum_{k=1}^N t_k} \prod_{k=1}^N (1 - e^{-qt_k})^{M-1},$$

and

$$l(t, q) = N \ln M + N \ln q - q \sum_{k=1}^N t_k + (M-1) \sum_{k=1}^N \ln(1 - e^{-qt_k}).$$

The unknown parameter can be calculated from

$$\frac{dl}{dq} = \frac{N}{q} - \sum_{k=1}^N t_k + (M-1) \sum_{k=1}^N \frac{t_k^{-qt_k}}{1 - e^{-qt_k}} = 0.$$

Having higher intensity rate for lower states is more reasonable assumption for degradation system models. Therefore let us consider the system with $q_i = (M+1-i)q$. In this model, the distribution function is:

$$\begin{aligned}
F(t) &= 1 - \sum_{i=1}^M \left(\prod_{j \neq i} \frac{(M+1-j)q}{q(j-i)} \right) e^{-(M+1-i)qt} \\
&= 1 - \sum_{i=1}^M \frac{M!}{(i-1)!(M-i)!(-1)^{M-i}} e^{-(M+1-i)qt} \\
&= 1 + \sum_{i=1}^M \frac{(-1)^{M-(i-1)} M!}{(i-1)!(M-(i-1))!} e^{-(M+1-i)qt}
\end{aligned}$$

Replacing $M - (i - 1)$ by k we get:

$$F(t) = 1 + \sum_{i=M}^1 \frac{(-1)^k M!}{(M-k)!k!} e^{-kqt} = (1 - e^{-qt})^M.$$

It is obvious that the obtained function is the same as that in the case $q_j = jq$. Consequently, the parameters estimators are the same ones.

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Р е з и м е

**ОЦЕНУВАЊЕ НА НАДЕЖНОСТА НА НЕПОПРАВЛИВ
ЕДНОКОМПОНЕНТЕН ХОМОГЕН СИСТЕМ СО ПОСТЕПЕНИ
РАСИПУВАЊА**

Во оваа статија разгледуваме повеќе состојбени еднокомпонентни системи со линеарно подредени нивоа на работа. Претпоставуваме дека во даден временски момент перформансите на системот можат да се намалат за едно ниво со константни интензитети на премин. При овие претпоставки пресметани се точните функции на веројатностите на премин кои потоа се користат за оценување на интензитетите на премин од една во друга состојба. Параметрите се оценети за два типа на системи, системи кои се под целосна контрола и системи за кои можеме да го набљудуваме само времето на целосното расипување на системот.

Клучни зборови: повеќеслоен систем; надежност; оценувачи

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